Name solutions

October 27, 2010

ECE 311 Exam 2 Fall 2010

Closed Text and Notes

1) Be sure you have 10 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) calculators are not allowed

4) Write neatly, if your writing is illegible then print.

5) Be sure you have the separate 4 pages that contain equations that may be of use to you.

6) This exam is worth 100 points.

(6 pts) 1. A stream of protons is flowing in free space all with the velocity $\mathbf{u} = -10\frac{\text{m}}{\text{s}}\mathbf{a}_z$. If the density of the protons is $2\text{x}10^{19}\ \frac{\text{protons}}{\text{m}^3}$, what is the current density?

$$\vec{J} = \rho\vec{u} = n e \vec{u}$$

$$= \left(2\times10^{19}\frac{\text{protons}}{m^3}\right)\left(1.6\times10^{-19}\frac{C}{\text{proton}}\right)\left(-10\frac{m}{s}\hat{a}_z\right)$$

$$= -32\frac{C}{m^2 s}\hat{a}_z$$

$$= -32\frac{A}{m^2}\hat{a}_z$$

(6 pts) 2. A parallel plate capacitor of plate separation 1 m is charged to 10 V. a hollow metal sphere of diameter 0.5 m is placed inside the capacitor between the plates. What is the electric field at the center of the sphere?

0

(5 pts) 3. Which of the following statements is correct?

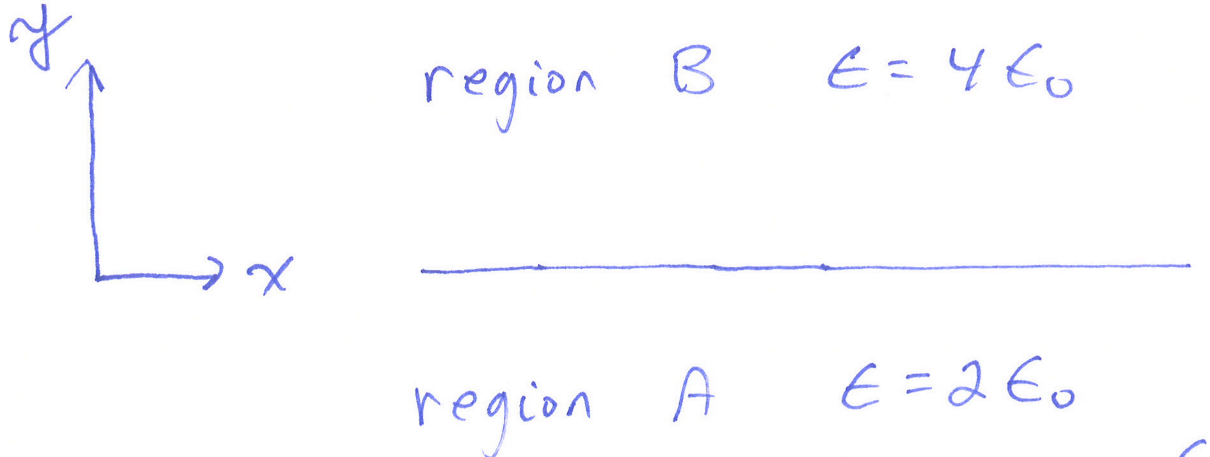
a) The electric flux density is due to free and bound charges, the electric field intensity is due to free charges, and the polarization is due to bound charges.

[✓ b)] The electric flux density is due to free charges, the electric field intensity is due to free and bound charges, and the polarization is due to bound charges.

c) The electric flux density is due to bound charges, the electric field intensity is due to free charges, and the polarization is due to free and bound charges.

d) The electric flux density is due to free charges, the electric field intensity is due to bound charges, and the polarization is due to free and bound charges.

(10 pts) 4. For region A, y < 0, $\varepsilon = 2\varepsilon_o$ and $\mathbf{D} = 4\ \hat{\mathbf{a}}_x - 6\ \hat{\mathbf{a}}_y + 8\ \hat{\mathbf{a}}_z\ \frac{C}{m^2}$. Find **D** in region B, where y > 0 and $\varepsilon = 4\varepsilon_o$.



$$\vec{D}_A = 4\hat{a}_x - 6\hat{a}_y + 8\hat{a}_z\ \frac{C}{m^2}$$

$$\vec{D}_{AN} = -6\hat{a}_y \frac{C}{m^2} = \vec{D}_{BN}$$

$$\vec{D}_{AT} = 4\hat{a}_x + 8\hat{a}_z\ \frac{C}{m^2}$$

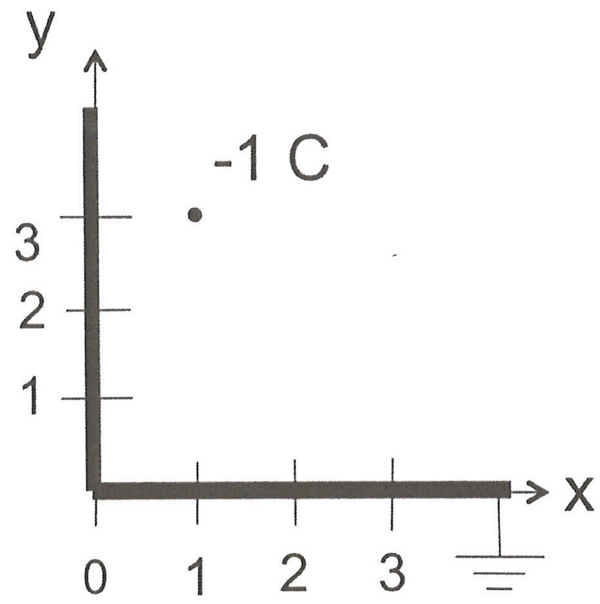
$$\vec{E}_{AT} = \frac{\vec{D}_{AT}}{2\epsilon_0} = \frac{2}{\epsilon_0}\hat{a}_x + \frac{4}{\epsilon_0}\hat{a}_z = \vec{E}_{BT}$$

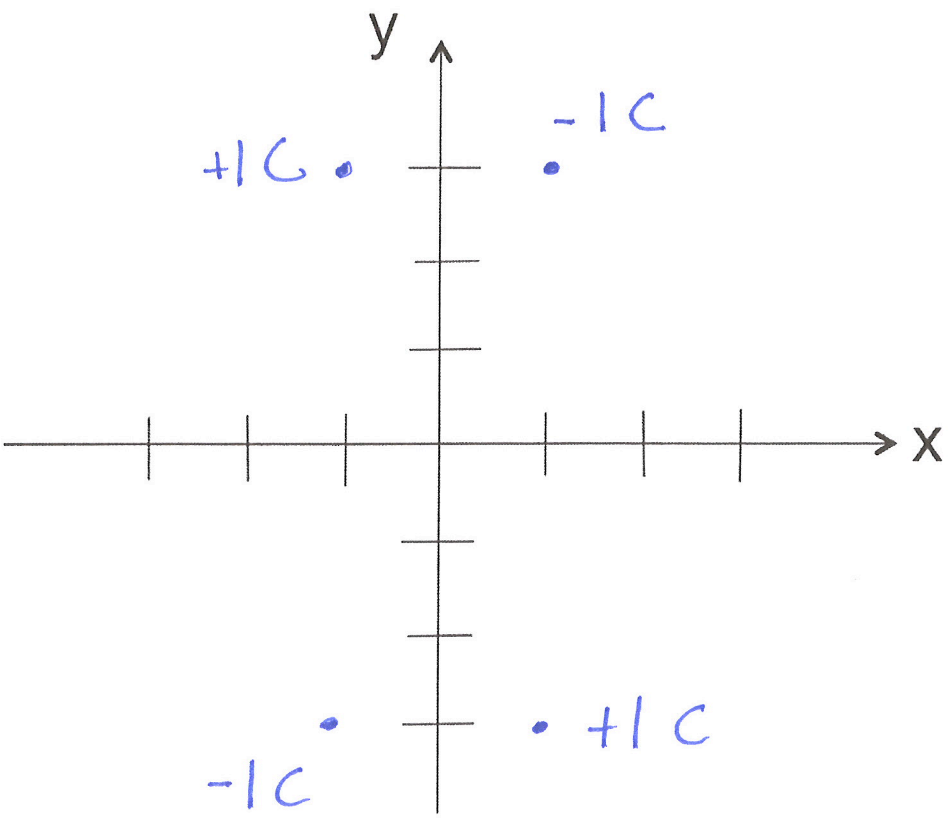
$$\vec{D}_{BT} = 4\epsilon_0 \vec{E}_{BT} = 8\hat{a}_x + 16\hat{a}_z \frac{C}{m^2}$$

$$\vec{D}_B = 8\hat{a}_x - 6\hat{a}_y + 16\hat{a}_z\ \frac{C}{m^2}$$

(5 pts) 5. A -1C point charge is located at (1, 3, 0) between two semi-infinite grounded planes as shown.

(3 pts) Show the system of image charges that could be used to find the voltage everywhere in the first quadrant.





(2 pts) What is the value of the electric field at (-1, -3, 0) for the system of the -1 C point charge and the two semi=infinite grounded planes?

0

(15 pts) 6. A parallel plate capacitor has plate area $1\ \text{m}^2$, plate separation $0.02\ \text{m}$, and a dielectric between the plates of relative permittivity $\varepsilon_r = 2$. A battery is connected to charge the positive plate with $8.854\text{x}0^{-10}\,\text{C}$ and the negative plate with $-8.854\text{x}0^{-10}\,\text{C}$. The plates are parallel to the yz plane and the positive plate is at x = 0 and the negative plate at x = ~~0.2~~ 0.02 m. Ignore field fringing at the edges of the capacitor.

(5 Pts) a) What is the electric flux density inside the capacitor?

$$\vec{D} = \frac{Q}{A}\,\hat{a}_x = \frac{8.854\times10^{-10}\,C}{1m^2}\,\hat{a}_x$$

$$\vec{D} = 8.854\times10^{-10}\,\hat{a}_x\,\frac{C}{m^2}$$

(5 Pts) b) What is the electric field intensity inside the capacitor?

$$\vec{E} = \frac{\vec{D}}{\epsilon_r \epsilon_0} = \frac{8.854\times10^{-10}\,\frac{C}{m^2}}{2\left(8.854\times10^{-12}\,\frac{F}{m}\right)}\,\hat{a}_x$$

$$\vec{E} = 50\,\hat{a}_x\,\frac{V}{m}$$

(5 Pts) c) What is the voltage drop across the capacitor

$$V = -\int_{0.02m}^{0} \vec{E}\cdot dx\,\hat{a}_x = -\int_{0.02m}^{0} 50\,\hat{a}_x\cdot dx\,\hat{a}_x\,\frac{V}{m}$$

$$= \left(-50\frac{V}{m}\right)\left(x\Big|_{0.02m}^{0}\right) = \left(-50\frac{V}{m}\right)\left(-0.02m\right)$$

$$= 1\,V$$

(10 pts) 7. A parallel plate capacitor has plate area $1\ \mathrm{m}^2$, plate separation 0.01 m, and free space between the plates. A 1 V battery is connected to charge the plates and then removed so as to not disturb the charge on the plates.

if the plates where pulled apart to 0.04 m without changing the charge on the plates,

(5 Pts) a) what would the voltage become?

The electric field would remain the same so $V = 4V$

(5 pts) b) How would the energy stored in the capacitor change?

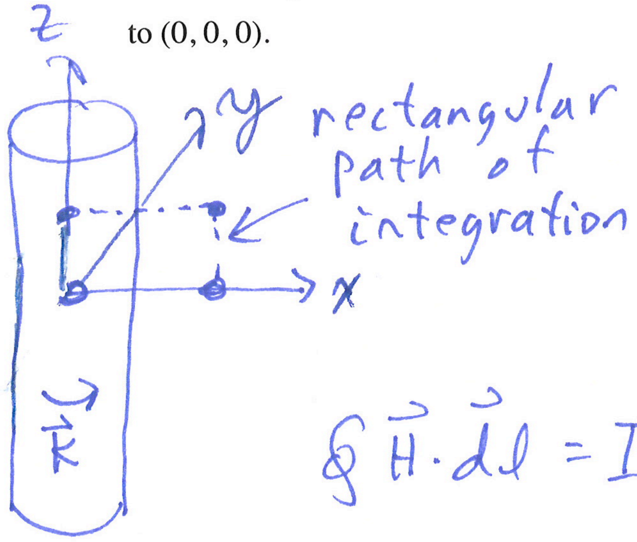
$W_E = \frac{1}{2} QV$ so the stored energy would increase by a factor of 4 because Q remains the same but V increases by a factor of 4

(6 pts) 8. Fill in the table with the standard units for the following

magnetic flux density, **B**	Wb/m^2
Magnetic field intensity, **H**	A/m
Electric Field Intensity, **E**	V/m
Electric Flux Density, **D**	C/m^2
Electric flux, Ψ	C
Magnetic flux, Ψ	Wb

(5 pts) 9. A hollow cylinder of radius 1 m has a sheet current density flowing on it of $\mathbf{K} = 10\,\frac{\text{A}}{\text{m}}\,\hat{\mathbf{a}}_\varphi$

Determine $\oint \mathbf{H}\cdot d\mathbf{l}$ for the rectangular path (0, 0, 0) to (2m, 0, 0) to (2m, 0, 1m) to (0, 0, 1m) to (0, 0, 0).



$$\oint \vec{H}\cdot d\vec{l} = I_{encl} = \left(-10\frac{A}{m}\right)(1m)$$

$$= -10A$$

(6prs) 10. A 100 m long wire of cross-sectional area $0.01\ \text{m}^2$ has 1 A flowing in it when a battery of 1 V is connected across it.

(3 pts) what is the electric field intensity in the wire?

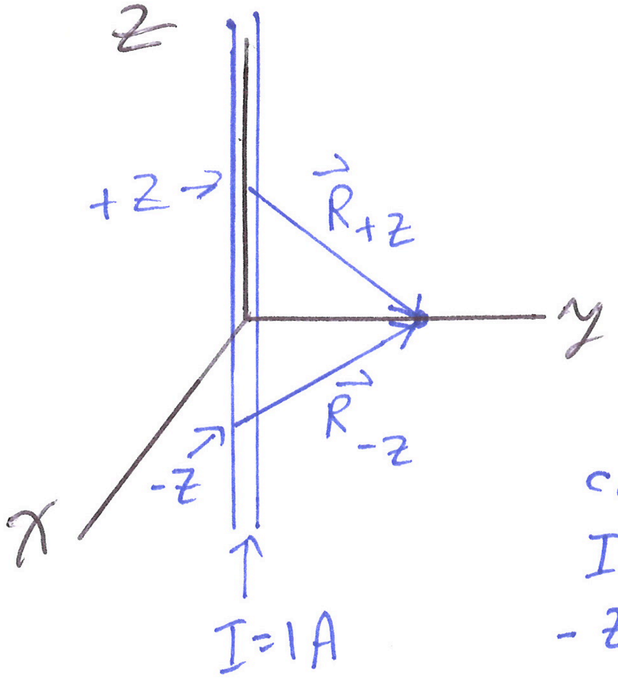
$$E = \frac{1V}{100m} = 0.01\frac{V}{m}$$

(3 pts) What is the he conductivity of the wire?

$$R = \frac{V}{I} = \frac{1V}{1A} = 1\,\Omega = \frac{1}{\sigma}\frac{L}{S} = \frac{1}{\sigma}\frac{100m}{0.01m^2}$$

$$\sigma = \frac{1}{1\Omega}\frac{100m}{0.01m^2} = 10^4\frac{1}{\Omega m} = 10^4\frac{S}{m}$$

(10 pts) 11. An infinitely long straight filament carries a current of 1A. Determine the magnetic field intensity everywhere.



From symmetry, $\vec{H}$ will not depend on ϕ or z. So,

$$\vec{H}(\rho)$$

Since $$\vec{dH} = \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2}$$

contributions to $\vec{dH}$ from $I\vec{dl}$ segments at $+z$ and $-z$ will result in only an $\hat{a}_\phi$ component. So,

$$\vec{H} = H_\phi(\rho)\hat{a}_\phi$$

Let's apply Ampere's circuital law with the path of integration being a circle of radius ρ centered on the z-axis and parallel to the xy plane.

$$\int_0^{2\pi} H_\phi(\rho)\,\rho\, d\phi = 1A$$

$$\rho H_\phi(\rho)\int_0^{2\pi} d\phi = 1A$$

$$\rho H_\phi(\rho)\,2\pi = 1A \longrightarrow H_\phi(\rho) = \frac{1A}{2\pi\rho}$$

$$\vec{H} = \frac{1A}{2\pi\rho}\hat{a}_\phi$$

(10 pts) 12. In Cartesian coordinates, the magnetic field intensity is $\mathbf{H} = x^2y\hat{\mathbf{a}}_y - y^2x\hat{\mathbf{a}}_z \ \frac{A}{m}$. Determine the current density at the point (2, 3, 4)

$$\vec{J} = \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x^2y & -y^2x \end{vmatrix}$$

$$= \hat{a}_x\left[\frac{\partial}{\partial y}(-y^2x) - 0\right] - \hat{a}_y\left[\frac{\partial}{\partial x}(-y^2x) - 0\right] + \hat{a}_z\left[\frac{\partial}{\partial x}(x^2y) - 0\right]$$

$$\vec{J}(x,y,z) = -2yx\,\hat{a}_x + y^2\,\hat{a}_y + 2xy\,\hat{a}_z \ \frac{A}{m^2}$$

$$\vec{J}(2,3,4) = -2(3)(2)\hat{a}_x + (3)^2\hat{a}_y + 2(2)(3)\,\hat{a}_z \ A/m^2$$

$$\vec{J}(2,3,4) = -12\,\hat{a}_x + 9\,\hat{a}_y + 12\,\hat{a}_z \ \frac{A}{m^2}$$

(6 pts) 13. An infinite plane at y = -4 m carries a sheet charge density of $20 \ nC/m^2$. Determine $\oint \mathbf{B} \cdot \mathbf{dS}$ over a sphere centered at the origin and of radius 1 m.

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